The Ignorant Observer inside Everett

A Finite-Record Quotient Theory

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Abstract

The Ignorant Observer Framework (IOF) treats collapse-like appearances and the experienced arrow of time as effects of finite observation. A finite observer does not have direct access to the global quantum state. It keeps records, tracks a physical reference frame, and loses operational control whenever the reference produces distinguishable structure faster than the observer can register it. The relevant rate is

$$\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2.$$

This paper places that idea inside Everettian quantum mechanics. Everett supplies the no-collapse host: Hilbert space, unitary dynamics, decoherence, phase, and the full Born-measure problem. The IOF supplies the finite-record quotient: the map from the host's projective state to the classical record simplex, the Fisher–Rao geometry of that simplex, the calibrated binary Born form, and the reference-tracking obstruction.

The paper has four main claims. First, decoherence realizes the modulus map on the observer's pointer records, so finite records see the probability simplex rather than the full projective state. Second, the Fisher–Rao metric on that simplex is both the quotient of the host's projective metric and the unique record geometry invariant under sufficient stochastic recodings; this yields the binary Born form under the stated calibration premises, but not the phase structure. Third, exact self-contained reference closure fails by Breuer-style non-factorization, and approximate tracking obeys the data-rate inequality $\kappa_{\mathrm{op}} \geq h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$. Fourth, the production rate h_{KS} is necessarily semiclassical: it exists only on the einselected branch domain, not as an intrinsic entropy production of a finite closed quantum system.

Everett [1] is therefore used as the preferred host, not as a theorem forced by the IOF. The operational law is host-independent and remains inside standard quantum mechanics. If the proposed BLQC and Fisher-homogeneity tests [2] succeed, they calibrate a finite-reference control law; they do not by themselves distinguish Everett from other no-collapse hosts.

1 Introduction

The Ignorant Observer Framework begins with a simple physical constraint [3]. A measurement basis is not an abstract label floating above the laboratory. It is implemented by a reference system–an apparatus orientation, phase standard, clock, calibration chain, or more generally a physical variable $\theta(t)$. A finite observer must track that variable through a finite channel. If the reference dynamics produces distinguishable alternatives at Kolmogorov–Sinai rate h_{KS}, and the observer can extract useful information at rate C_{eff}, the deficit is

$$\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2.$$

When this quantity is positive, no finite tracking scheme can keep the reference error bounded indefinitely. Before saturation the unresolved basis variance grows at least exponentially, and the observed interference contrast acquires the recoverable factor

$$V_{\mathrm{IOF}} = \exp(-\sigma_\theta^2/2).$$

The characteristic empirical sign is a reversal relative to ordinary thermal decoherence: increasing the observer's tracking capacity should extend coherence rather than shorten it [2].

The BLQC benchmark [2] is deliberately modest about what such a result would show. In the engineered regime its κ-scaling is also the scaling of an ordinary classical controller tracking an injected disturbance; the test calibrates a reference-channel law inside standard quantum mechanics rather than separating Everett from other no-collapse hosts. The separate regime in which information extraction itself feeds back as physical instability is treated in the capacity–backaction analysis [4].

That statement is operational. It is a claim about a reference loop within standard quantum mechanics. It does not require an interpretation of the quantum state. The larger question is interpretive: if the underlying dynamics is deterministic and no-collapse, why does a finite observer report one definite quasi-classical history with Born weights? This paper develops the IOF answer in an Everettian host.

The answer is not that the IOF replaces Everett. It does not. Everett supplies the global quantum state, unitary dynamics, decoherence, relative phase, and the unresolved global measure problem. The IOF describes what a finite embedded observer can record from that host. Its object is the finite-record quotient: the projection from the host's projective state space to the observer's classical record algebra.

Stated as a question, the paper asks:

> *Why does a unitary no-collapse substrate appear, to a finite embedded observer, as a single world of definite records governed by Born weights?*

The proposed answer is layered. Decoherence produces stable pointer records. The finite observer reads only the modulus data of those records, not the full phase structure. The resulting record simplex carries Fisher–Rao geometry. Under a calibration premise this geometry gives the binary Born form [5]. The reference basis itself cannot be closed exactly from the observer's own records, and approximate tracking is limited by the data-rate inequality above.

Scope. The paper keeps four distinctions explicit.

1. The operational core is host-independent. The rate κ, the sign-reversal signature, the recoverable visibility channel, and the finite-record geometry do not depend on choosing Everett.
2. Everett is the preferred host here because it is the cleanest no-collapse setting: it needs no added collapse dynamics and evades the Bell dilemma by rejecting single outcomes rather than measurement independence. This is a preference, not a proof of exclusivity.
3. The Born result is a form result. It derives the calibrated binary weight $\cos^2(\theta/2)$ from record geometry under stated premises [5]. It does not by itself derive the existence of the full branch measure, its cross-context coherence, or its rational use.

4. The production rate h_{KS} is not a closed finite-dimensional quantum invariant. It belongs to the einselected semiclassical branch dynamics. The framework is therefore a unitary host plus semiclassical record-dynamics theory, not a derivation from a finite closed quantum system alone.

The rest of the paper follows the architecture. Section 2 states the record quotient. Sections 3 and 4 separate the Born form on records from the phase structure kept by the host. Section 5 proves the reference-closure obstruction and the data-rate inequality. Section 6 explains why h_{KS} is necessarily semiclassical. Sections 7–9 discuss the movable cut, probability at the record boundary, and the division of labour among hosts. The appendices give the elementary proofs and the rate converse.

2 The Layered Architecture

The host owns the global quantum state. The observer owns only a finite record. The useful way to read the Everett–IOF construction is therefore as a stack, with the unitary substrate at the bottom and the finite constructor at the top.

At the base is the host state space $\mathbb{CP}^{n-1}$, with Fubini–Study/Kähler geometry and unitary evolution. Decoherence and partial trace give the reduced pointer state, approximately diagonal in an einselected basis. A finite readout then maps this state to a probability vector in the simplex Δ_n. On that simplex, sufficient stochastic recodings select the Fisher–Rao metric. Finally, the observer tracks the physical reference with finite capacity, producing the rate $\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$.

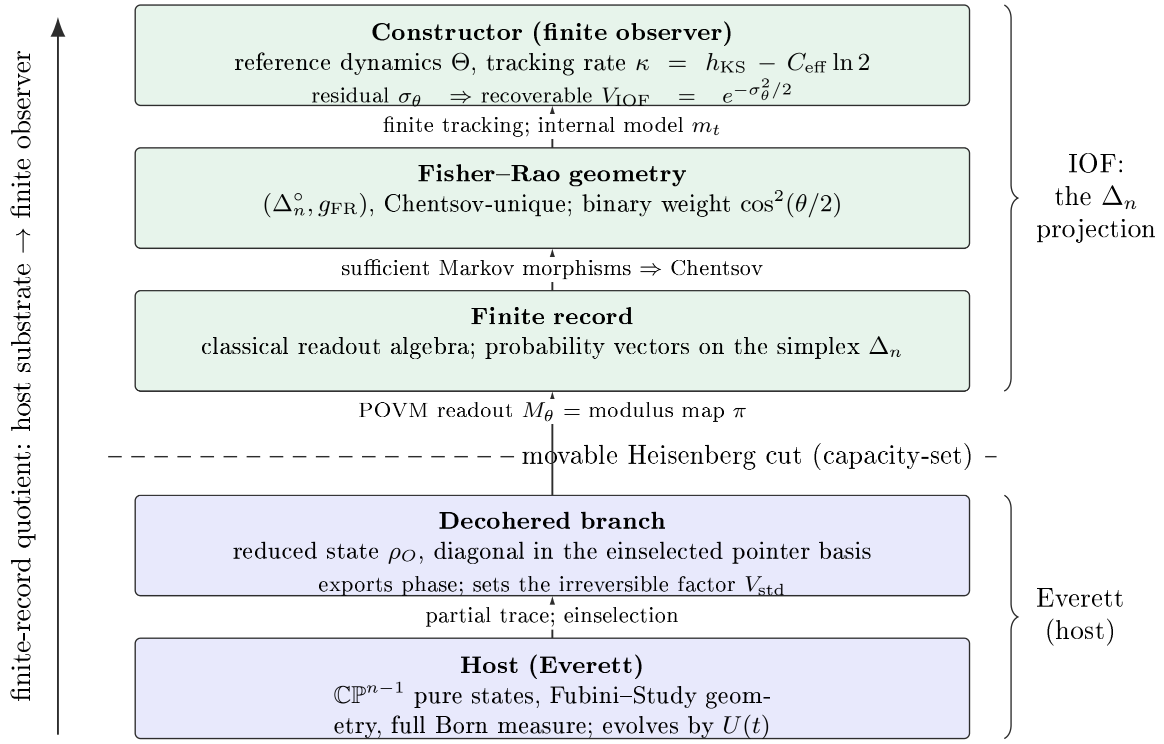

Figure 1: The Everett–IOF stack. The host provides the projective quantum state and unitary dynamics. Decoherence and finite readout give a classical record in Δ_n. The IOF studies that record quotient: its Fisher–Rao geometry and the finite tracking law $\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$. The observed visibility factorizes, when the environmental and reference errors are independent, as $V_{\mathrm{obs}} = V_{\mathrm{std}} V_{\mathrm{IOF}}$.

The map from host state to record base is the modulus map.

Definition 2.1 (Modulus map). In the instantaneous basis $\{|i_\theta\rangle\}_{i=1}^n$, define

$$\pi_\theta : \mathbb{CP}^{n-1} \longrightarrow \Delta_n, \qquad [\psi] \longmapsto \big(|\langle i_\theta|\psi\rangle|^2\big)_{i=1}^n.$$

It keeps the squared moduli and forgets the relative-phase fibre.

Proposition 2.2 (Decoherence realizes the record quotient). *Let $K_t : \Delta_n \to \Delta_m$ be the Markov kernel describing the observer's finite coarse-grained readout. Suppose the measurement–decoherence step produces environmental records with $|\langle E_i(\theta)|E_j(\theta)\rangle| \leq \varepsilon$ for $i \neq j$. Then the finite record distribution satisfies*

$$R_t([\psi]) = K_t \circ \pi_{\theta_t}([\psi]) + O(\varepsilon)$$

in total variation. More explicitly,

$$\big\|R_t([\psi]) - K_t\pi_{\theta_t}([\psi])\big\|_1 \leq C\varepsilon,$$

where C depends on the readout algebra but not on $[\psi]$.

Proof. After decoherence the observer's reduced pointer state is diagonal in the pointer basis up to off-diagonal terms controlled by the environment overlaps:

$$\big\|\rho_O - \operatorname{diag}\pi_{\theta_t}([\psi])\big\|_1 \leq C'\varepsilon.$$

The finite readout, including K_t, is a completely positive trace-preserving map and hence contracts trace distance. This gives the stated bound. The phase fibre does not enter the diagonal record, so the record depends only on $\pi_{\theta_t}([\psi])$. □

Remark 2.3 (Decoherence exports phase). The host remains unitary. Decoherence does not destroy relative phase; it exports phase into system–environment correlations and makes it locally inaccessible. The correct statement is that decoherence realizes the modulus map on the observer's pointer-record algebra.

Remark 2.4 (Two readings of the reduced state). The unconditioned reduced state

$$\rho_O \simeq \sum_\alpha |c_\alpha|^2 |O_\alpha\rangle\langle O_\alpha|$$

gives branch weights. A branch-conditioned state $\rho_O^{(\alpha)}$ gives future readout within one branch. The same density-matrix algebra is used in both readings; the interpretation is different.

Remark 2.5 (The record simplex is an interface). The framework does not assume records from nowhere. Records are produced by decoherence, coarse-graining, and finite capacity. Once a stable pointer-record algebra exists, its mutually exclusive record alternatives are represented by normalized additive weights, a point of Δ_n. The IOF derives constraints on the form of these weights. It does not derive the existence of the host's full branch measure from nothing.

The accessible record algebra is also not arbitrary. It is fixed by what the environment redundantly records and what the observer can capture.

Assumption 2.6 (SBS capture). After the measurement–decoherence step, the pointer and the captured environment fragment f are $(\varepsilon_s, \varepsilon_f)$-approximately spectrum-broadcast in the einselected pointer basis. That is, there is a state

$$\sigma = \sum_i p_i |i\rangle\langle i| \otimes \bigotimes_k \sigma_i^{(k)}$$

with $\frac{1}{2}\|\rho - \sigma\|_1 \leq \varepsilon_s$ and pairwise fidelities $F(\sigma_i^{(k)}, \sigma_j^{(k)}) \leq \varepsilon_f$ for $i \neq j$. Exact SBS is recovered when $\varepsilon_s = \varepsilon_f = 0$ [6–8].

Proposition 2.7 (Accessible record algebra, SBS-conditional). *Under Assumption 2.6, with record capacity m and n pointer values:*

1. *The broadcast conditional structure generates a commutative record algebra A_f, readable by the pretty-good measurement with single-shot error $\delta \leq (n-1)\varepsilon_f + \varepsilon_s$. The induced disturbance of the pointer and any disjoint fragment is at most $2\sqrt{\delta}$.*
2. *The observer's accessible algebra A_O is the coarse-graining of A_f to at most 2^m elements.*
3. *Observers on disjoint fragments agree on the pointer index up to the corresponding readout errors; for t observers with error $\leq \delta$, joint agreement holds with probability at least $1 - t\delta$.*
4. *In the exact SBS case, any redundantly readable, non-disturbing fragment observable is contained in A_f up to relabeling.*

Proof. See Appendix C. The proof uses the Barnum–Knill pretty-good-measurement bound, the Fuchs–van de Graaf fidelity–distance estimates, the gentle measurement lemma, and the corrected strong-Darwinism/SBS equivalence [8–14]. □

Remark 2.8 (Accessible algebra, not absolute factorization). The construction does not require a unique fundamental tensor factorization of the universe. It requires, for each finite observer, a physically selected accessible record algebra: the stable, decohered, redundant, finite-capacity part of the host state the observer can actually use. This algebra is indexed by the captured fragment and memory capacity. It is movable, but not arbitrary.

2.1 Observer, branch, and experience

An *observer* has two descriptions. In the host it is a stable pattern with redundant records, in Wallace's sense [15] and in Zurek's decoherence/Darwinism sense [16]. In the IOF it is the same physical subsystem priced operationally by C_{eff}, h_{KS}, and κ. The host says what record structure exists; the IOF says what it costs a finite system to form and maintain it.

A *branch* is not a primitive added universe. It is an approximately autonomous, decohered, redundantly recorded sector of the one global state. The word "world" is convenient only after indexing it to a record algebra. Unindexed, the ontology is the host state and its correlations.

An *experience*, for this paper's purposes, is a branch-local update of a physical memory record. This is not a theory of consciousness. It is the weaker claim that anything an embedded observer can report must be encoded as a finite physical record. The observer never reports the global superposition as an accessible object; it reports from within a decohered record sector.

3 Bridge 1: Record Geometry and the Binary Born Form

The first bridge relates finite records to the Born form. It has two parts. First, the record geometry is the modulus part of the host's projective geometry. Second, under a calibration premise, that geometry gives the binary Born weight.

Write

$$|\psi\rangle = \sum_i \sqrt{p_i}\, e^{i\phi_i} |i\rangle.$$

The Fubini–Study line element splits as

$$g_{\rm FS} = \tfrac{1}{4} g_{\rm FR} + g_{\rm phase}.$$

Indeed,

$$ds^2_{\rm FS} = \frac{1}{4} \sum_i \frac{dp_i^2}{p_i} + \left(\sum_i p_i d\phi_i^2 - (\sum_i p_i d\phi_i)^2 \right).$$

On diagonal density matrices the Bures metric restricts to

$$ds^2_{\rm Bures} = \frac{1}{4} \sum_i \frac{dp_i^2}{p_i} = \frac{1}{4} ds^2_{\rm FR}.$$

Thus Fisher–Rao is not imported ad hoc. It is the host's projective metric after quotienting the phase fibre. The same metric is also selected intrinsically by Chentsov's theorem as the unique metric on finite probability distributions invariant under sufficient Markov morphisms [17, 18]. The host restriction and the record invariance pick out the same geometry.

For a binary record, introduce square-root coordinates $q_i = \sqrt{p_i}$. The simplex becomes a sphere, and Fisher arclength gives

$$p(s) = \cos^2(s/2).$$

If the physical basis coordinate θ is calibrated so that equal increments of θ cost equal Fisher distinguishability, then $s = \alpha\theta$. The endpoint conditions $p(0) = 1$ and $p(\pi) = 0$ on the first monotone interval fix $\alpha = 1$. Hence

$$p(\theta) = \cos^2(\theta/2).$$

Theorem 3.1 (Conditional binary Born form, after [5])**.** *Under the Fisher capacity bridge, finite-resolution invariance under sufficient Markov maps, scalar-threshold homogeneity of the calibrated basis coordinate, and monotone endpoint calibration, the calibrated two-outcome record has Born weight*

$$p(\theta) = \cos^2(\theta/2).$$

The theorem should be read narrowly. It derives the binary *form*; it does not derive probability existence. It begins once there is a finite record algebra whose mutually exclusive alternatives are represented by normalized additive weights. The quantitative identity is Wootters's statistical-distance result in this two-outcome setting [19]. The IOF contribution is the proposed physical meaning of the calibration: finite reference tracking preserves operational distinguishability, so the natural coordinate is Fisher homogeneous. That premise is exposed to the Fisher-homogeneity module of the BLQC benchmark [2].

Nor is the result a fix for a $d = 2$ Gleason gap. "Binary" means two-outcome record, not two-dimensional Hilbert space. POVM-Gleason theorems already reach $d = 2$ by assuming the full effect algebra [20, 21]. The records route assumes less quantum structure and reaches a weaker, fixed-context conclusion.

Remark 3.2 (Multi-outcome contexts). The extended companion [5] treats finite multi-outcome records by hierarchical dial-local calibrations. For any single measurement context it recovers $p_i = q_i^2$ under the corresponding conditional premises. What remains outside the record route is cross-context coherence and phase.

Deutsch–Wallace. The Everettian probability problem separates into three questions: existence of a measure, form of the weights, and rational use of those weights. The IOF addresses the middle question for calibrated record contexts. This narrows, but does not eliminate, the Deutsch–Wallace burden [15, 22, 23]. Decision theory is no longer asked to manufacture the binary squared-amplitude form. It is still asked to justify the rational use of the measure, especially before branching.

4 The Phase Limit and the Born Split

The record bridge stops at the modulus. It does not recover phase. This is not merely a missing construction; it is a structural limit.

Theorem 4.1 (Phase limit, after [24])**.** *On finite simplices Δ_n°, invariant 2-tensors under sufficient Markov morphisms form a one-dimensional space spanned by the symmetric Fisher metric. The antisymmetric part of any invariant 2-tensor vanishes, and the only invariant* $(1,1)$*-tensor is a scalar multiple of the identity. Hence finite-record geometry has no invariant almost-complex, symplectic, or Kähler structure.*

The finite record keeps squared moduli. It forgets the phase fibre. The missing complex structure is the differential expression of that quotient. Therefore the IOF can recover the single-context weight form, but not interference between incompatible contexts. Phase and cross-context coherence remain host-side.

Record side	**Host side**
single-context weights, including calibrated binary $\cos^2(\theta/2)$	relative phase and interference
$\frac{1}{4}g_{\mathrm{FR}}$, the modulus metric	the phase-fibre metric and Kähler structure
finite-record distinguishability	non-commuting context composition
record-level probability calculus	cross-context coherence and rational use of the full branch measure

This is the Born split used throughout the paper. The IOF gives a record-geometric route to the amplitude-squared form within a context. The host supplies the phase structure that makes the contexts quantum.

5 Bridge 2: Reference Closure and the Data-Rate Inequality

The second bridge concerns the reference itself. The basis variable θ_t is a physical degree of freedom of the observer's apparatus and environment. Since the observer is part of the system it tries to track, two questions arise. Can the observer recover the reference exactly from its own records? If not, how fast must approximate tracking fail?

5.1 Exact closure fails by non-factorization

It is tempting to say that exact self-modelling is impossible because the model would have to contain itself. That slogan is wrong. Self-reference is not automatically inconsistent. The exact obstruction is instead a factorization failure.

Theorem 5.1 (Exact closure fails by non-factorization, after [25])**.** *Let $R_{\leq t}$ be the observer's finite record history and Θ_t a basis functional on global histories. Exact self-contained reference closure–a function F_t such that $\Theta_t = F_t \circ R_{\leq t}$–fails if and only if there are admissible histories x, y with*

$$R_{\leq t}(x) = R_{\leq t}(y), \qquad \Theta_t(x) \neq \Theta_t(y).$$

Proof. This is the fibre-constancy criterion. Appendix A gives the set-theoretic proof; Breuer's self-measurement result supplies the physical instance. □

The point is exact, not statistical. Two different global states can give the same local observer state and the same local statistics while differing in a global variable relevant to the reference. No increase of local processing power can invert a non-injective restriction.

5.2 Approximate tracking obeys a data-rate bound

Once exact closure fails, the observer can only track approximately. The rate question is then classical in form. Distinguishable reference histories grow like $e^{h_{\mathrm{KS}} T}$. The observer's channel can distinguish at most $e^{C_{\mathrm{eff}} \ln 2\, T}$ histories. If production outruns extraction, histories collapse onto the same internal records and bounded tracking fails.

The static coordinate in Bridge 1 and the dynamical coordinate here are connected by an explicit modelling premise.

Assumption 5.2 (Basis–reference identification)**.** The calibrated basis coordinate θ on the record simplex and the tracked reference coordinate θ_t are the same physical variable: a smooth coordinate of the observer's reference dynamics, advected along the unstable manifold in the semiclassical regime of Assumption B.1. This is the measurement-basis premise argued in [26]; it is assumed here.

Let $\kappa_{\mathrm{def}} = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$. The operational growth rate is measured before saturation by

$$\kappa_{\mathrm{op}}(t) = \frac{1}{2t} \ln\left(\frac{\sigma_\theta^2(t)}{\sigma_{0,\mathrm{eff}}^2}\right).$$

On a compact physical coordinate this statement is read up to the first time σ_θ reaches the chart scale. On the universal cover no saturation qualifier is needed.

Theorem 5.3 (Inequality form)**.** *Assume the within-branch reference dynamics is, in the semiclassical limit, a smooth uniformly hyperbolic map with an SRB measure. Then*

$$\kappa_{\mathrm{def}} = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2 \geq h_{\mathrm{KS}} - \ln d_O$$

and, throughout the pre-saturation tracking window,

$$\kappa_{\mathrm{op}} \geq h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2 = \kappa_{\mathrm{def}}.$$

Thus, if $h_{\mathrm{KS}} > \ln d_O$ per extraction step, an embedded observer of dimension d_O cannot keep bounded tracking error of its own reference.

Proof. The structural bound is the Holevo ceiling $C_{\text{eff}} \ln 2 \le \ln d_O$. The operational bound is the typical-case estimation converse proved in Appendix B, applied to the unstable coordinate with SRB production rate h_{KS}. Pesin's identity supplies $h_{\text{KS}} = \sum_i \lambda_i^+$ on the same measure class. ☐

Only the inequality is claimed. Equality would require a generating partition and an estimator saturating the channel. Generic observers have inefficiencies, non-generating records, and noise; these make the inequality strict. The BLQC benchmark therefore fits an effective calibrated rate rather than assuming exact identity [2].

6 The Production Rate Is Semiclassical

A finite closed quantum system evolves unitarily and has no intrinsic asymptotic entropy production. This matters because the IOF rate law uses h_{KS}, a classical dynamical entropy. The production rate must therefore be located carefully.

Theorem 6.1 (Emergent reference rate)**.** *A closed physical system with bounded energy and finite volume has zero intrinsic asymptotic quantum dynamical entropy. The CNT and ALF entropies vanish for finite-dimensional automorphisms, and bounded energy in finite volume gives a discrete spectrum, hence quasi-periodic non-mixing dynamics. A sustained positive rate* h_{ref} *becomes available only in a spectrum-continuing limit: the semiclassical limit, the open continuous-measurement limit, or the thermodynamic limit. In the einselected semiclassical branch domain,* $h_{\text{ref}} = \sum_i \lambda_i^+$*, the Pesin sum.*

Argument. For finite-dimensional closed automorphisms, CNT and ALF dynamical entropies vanish [27, 28]. Bounded energy and finite volume yield discrete spectrum by Weyl's law, so closed dynamics is quasi-periodic rather than mixing. Positive sustained entropy production requires a continuous-spectrum or limiting description. In the monitored chaotic regime, the Zurek–Paz result gives entropy production at the classical Lyapunov/Pesin rate [29, 30]. ☐

The framework is therefore a hybrid by necessity.

Remark 6.2 (Hybrid by necessity). The IOF is not a pure consequence of finite closed unitary dynamics. It is a unitary no-collapse host plus an einselected semiclassical record dynamics. That is not a temporary gap; it is the domain on which the production rate is defined. The finite observer itself is an open, monitored, record-forming system, so this is also the domain in which the framework naturally lives.

Lyapunov exponents and Kolmogorov–Sinai entropy should not be conflated. A Lyapunov exponent is a tangent-space divergence rate. h_{KS} is an information-production rate. They coincide as a Pesin sum under SRB hypotheses; otherwise one has only inequalities such as Ruelle's $h_\mu \le \sum_i \lambda_i^+$ [31, 32].

7 Decoherence, Two Visibility Channels, and the Movable Cut

Decoherence is host-side physics. It selects pointer records and exports phase to the environment. The IOF adds a second loss channel: finite uncertainty about the reference. When the two smearings are statistically independent,

$$V_{\text{obs}} = V_{\text{std}}\, V_{\text{IOF}}.$$

The factor V_{std} is ordinary environmental decoherence. It is reversible only by recohering the relevant environment, which is practically unavailable. The factor $V_{\text{IOF}} = \exp(-\sigma_\theta^2/2)$ is reference averaging. It is recoverable whenever the realized reference can be logged and used in post-processing. This is standard reference-frame physics, not a new collapse mechanism [33].

Under a unitary host both channels are recoverable in principle. Objective-reduction theories deny this. The BLQC recoverability classifier R_{rec} tests that distinction: a genuine unrestorable drop in a regime where the unitary host predicts recoverability would count against no-collapse dynamics and in favour of collapse-type physics [2, 34]. It would not distinguish Everett from other no-collapse hosts.

The same rate κ sets the movable Heisenberg cut. Where $\kappa_{\text{def}} > 0$, finite tracking fails and the observer must treat the reference as a classical record with residual uncertainty. Where the deficit is absent, more of the system can remain coherently tracked, subject to estimator quality. The cut is therefore observer-indexed and capacity-set: movable but physically constrained.

8 Probability at the Record Boundary

The IOF does not make objective chance fundamental. Probability appears at the record boundary, where a finite observer must assign weights to mutually exclusive record alternatives. This section states how much of that structure can be obtained without using the full Everettian decision-theoretic machinery.

8.1 Existence from bookkeeping coherence

Proposition 8.1 (Existence at the record boundary)**.** *Let A be a finite record algebra. Let $\beta : A \to [0, 1]$ be the observer's fair rates for unit bets on record events. Suppose the rates are tower-consistent: sufficient refinements and coarse-grainings of records preserve the rates by pushforward. Then β is weakly coherent–no finite two-sided portfolio has uniformly negative payoff–if and only if β is a finitely additive normalized measure on A. Tower-consistent families form compatible measure systems under the connecting recodings.*

Proof. See Appendix D. It is the finite de Finetti coherence argument with the tower condition added as a pushforward condition [35]. One-sided betting would give lower and upper previsions rather than sharp probabilities [36]. □

The same conclusion can be reached by accuracy arguments: credences violating the probability axioms are strictly dominated under strictly proper scoring rules [37]. The point is modest but important. The record simplex is not merely inherited from the host; coherent bookkeeping on finite records already has the measure form.

Remark 8.2 (Route robustness)*.* The measure conclusion does not depend on the betting idiom. The operational coherence route and the accuracy-domination route land on the same finite measure structure. Together with the independent geometric route to the calibrated Born form in Section 3, this helps answer the worry that the record-credence principle has simply been tailored to deliver Born weights: the probability calculus, the fixed-context form, and the self-locating use claim enter through different premises.

Corollary 8.3 (No branch counting)**.** *Under tower-consistency, branch-counting credences are incoherent. A sufficient refinement that splits a record event without changing any record content changes the count but must not change the pushed-forward rate.*

8.2 Self-location after branching

Proposition 8.4 (The self-locating window). *Let D_t be the algebra of decohered branch distinctions by time t, and let $R_t \subseteq D_t$ be the subalgebra registered in the observer's accessible records. Then registered information grows by at most C_{eff} bits per unit time. A branching event of pointer entropy H opens a window of duration at least H/C_{eff} during which several decohered successors restrict identically to the observer's record algebra. If production persists with $h > C_{\mathrm{eff}} \ln 2$, the unregistered entropy grows without bound.*

Proof. The record channel carries at most C_{eff} bits per unit time. Until the mutual information with the pointer reaches H, the registered algebra cannot distinguish all decohered alternatives. Subtracting rates gives the persistent case. □

During this window the observer is in the self-locating situation discussed by Vaidman, Sebens–Carroll, and McQueen–Vaidman [38–40]: there are several successors with identical accessible evidence.

Assumption 8.5 (Record-credence principle). Rational credence is a functional of the accessible record state. Physical operations that preserve the accessible record state and the record-identity structure of the alternatives leave credences unchanged; isomorphisms of the record embedding merely relabel credences. This is a substantive indifference principle restricted to record-identical alternatives.

Proposition 8.6 (Born credences for record-identical branches). *Assume the record-credence principle, tower-consistent coherence, and the host-side resources of decohered branching, record-external swaps, and fine-graining. Then self-locating credences over record-identical branches equal the Born weights:*

$$c_k = |\alpha_k|^2.$$

On calibrated record contexts these agree with the record-geometric weights.

Proof. See Appendix D. The proof is the standard equal-weight swap and fine-graining argument, but restricted to a single record-identical configuration. The host supplies the swap and fine-graining kinematics; probability enters through record coherence and the record-credence principle. □

Remark 8.7 (Partial registration). After partial readout, the record algebra partitions the decohered branches into equivalence classes. Proposition 8.6 applies within each class, and the total credence factorizes as the registered posterior over classes times Born self-location inside the class. Registration therefore interpolates continuously between self-location and ordinary classical updating.

This closes only the post-branching self-location half. It does not solve the ex-ante decision problem: why an agent before branching should use the weights in planning and betting. That diachronic bridge is inherited from the Everettian decision and self-location literature, and it carries much of the operational weight of probability.

Remark 8.8 (The Dawid–Friederich critique). Dawid and Friederich argue that the Sebens–Carroll epistemic-separability principle lacks an independently motivated conditioning object, mishandles branch multiplication cases, and functions more like a theory-specific physical posit than a general norm of reasoning [41, 42]. The present paper answers only part of that critique. The conditioning object is not chosen for convenience: it is the accessible record algebra selected by decoherence, broadcast structure, and finite capacity. Branch-counting alternatives are excluded by tower-consistency. And the route robustness of Remark 8.2 blocks the strongest "tailored to deliver Born" reading: coherence, accuracy, record geometry, and self-location do not all use the same premise. But the record-credence principle remains a substantive indifference norm, and the ex-ante bridge remains open. The claim here is a physically anchored post-branching identification, not a complete decision theory.

8.3 The normative core

After these reductions, the remaining normative content is small but real: weak coherence (or accuracy non-domination), tower-consistency, record-functionality, record-indifference, and the diachronic ex-ante decision bridge. The IOF does not pretend these are theorems of physics. It identifies where the norms enter and separates them from the geometric and dynamical claims.

Remark 8.9 (Time at the record boundary)*.* The arrow of time is also record-indexed. Untrackable loss alone has no direction: for invertible dynamics, Kolmogorov–Sinai entropy is time-reversal invariant. A direction requires a local entropy gradient so records can form, persist, and be overwritten at Landauer cost. Given that gradient, finite observation supplies the rest: the past is fixed because registered records are nested; the future is open because unregistered branch distinctions are not yet records; the present has a finite width set at least by H/C_{eff}; and the felt irreversibility is governed by the same tracking deficit. The framework presupposes dynamics; it derives the observer's arrow from record formation.

9 Hosts and the Division of Labour

The IOF is host-pluralist in its operational core. The host supplies the ontology beneath the records; the IOF supplies the finite-record quotient and the tracking law.

Everett. Everett is the preferred host in this paper. It keeps unitary dynamics, takes decoherence seriously, and avoids Bell's single-outcome dilemma by rejecting the single global outcome rather than measurement independence. Within Everett the IOF describes the branch-local finite observer: why reports are single-branch, why the record geometry has the binary Born form, and where the capacity-set cut lies. The cost is that Everett's own probability and global-structure questions remain.

Single-history hosts. In invariant-set or cellular-automaton style hosts [43, 44], κ does different work. It becomes a concealment rate for measurement dependence: the setting–system correlation may be part of the common history, but finite observers cannot reconstruct or exploit it from within. This addresses the detectability form of the superdeterminism objection. It does not by itself supply the underlying dynamics.

Pilot-wave hosts. In a Bohmian host, the IOF is an epistemic layer over definite hidden variables. The ontology gives single outcomes; finite records and reference tracking give the observer's effective probabilities. The cost is the usual nonlocality and a less natural fit to the shared-ancestry concealment story.

The closures of Section 8 fit Everett most naturally because swaps and fine-grainings are kinematic operations on the actual branching state. In a pilot-wave host the same reasoning is closer to a coherence route to quantum equilibrium. In single-history hosts it becomes a constraint on admissible-history measures rather than a native self-location derivation. This is why Everett is preferred here, while host-independence is retained as an operational hedge.

10 What This Does and Does Not Prove

The framework is closed only within its stated scope. Conditional on a no-collapse host and on the einselected semiclassical branch domain, the record quotient, phase limit, Breuer obstruction, data-rate inequality, and emergent-rate theorem fit together. It is not a self-standing replacement for quantum theory.

It relocates part of probability to the record boundary. Coherent bookkeeping gives a finite additive measure on records. Record geometry gives the fixed-context Born form. The record-credence principle plus host symmetries identifies post-branching self-locating credence with the Born weights. Cross-context coherence, phase, and the ex-ante decision bridge remain host-side.

It does not derive h_{KS} as a closed-quantum invariant. The production rate is semiclassical by theorem. This is a domain statement, not an unfinished derivation.

It does not prove the exact identity $\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$. The proved result is the lower bound on operational error growth. Equality requires additional achievability assumptions.

Its experiments are unrun. The sign-reversal, recoverability, and Fisher-homogeneity modules [2] are falsification channels for the operational law. Success would calibrate the finite-reference control law within standard quantum mechanics. It would not by itself choose between empirically equivalent no-collapse interpretations.

11 Objections and Replies

Remark 1 (Is this just decoherence relabelled?). No. Decoherence is the host-side process that produces pointer records and exports phase. The IOF adds the reference-tracking channel, with rate κ, recoverable visibility factor V_{IOF}, and sign-reversal signature.

Remark 2 (Is the Born result circular?). It assumes finite record alternatives with normalized additive weights and derives the calibrated binary form. It does not derive probability existence from nothing, and it does not derive the full cross-context Born measure. The circularity charge is avoided only by keeping that scope explicit.

Remark 3 (Does the paper overcommit to Everett?). No. Everett is the preferred host for this presentation. The operational core can be hosted elsewhere. Experimental success of the IOF modules would support the reference-channel law, not prove Everett uniquely.

Remark 4 (Is the semiclassical interface a gap?). No. A finite closed quantum system has no sustained intrinsic h_{KS}. The rate exists on the einselected semiclassical domain. That is the correct domain of the framework.

Remark 5 (Does this fill a $d = 2$ Gleason gap?). No. The result concerns two-outcome records, not Hilbert-space dimension two. POVM-Gleason theorems already cover $d = 2$ under stronger assumptions. The IOF route is parallel and weaker in scope.

Remark 6 (Does the self-reference claim contradict Kleene?). No. The obstruction is not self-reference as such. It is non-factorization: the global reference functional does not factor through the observer's finite local records.

12 Conclusion

Inside Everett, the Ignorant Observer Framework is a finite-record quotient theory. The host supplies the global state, phase, decoherence, and the full Born-measure machinery. The IOF supplies the record-level geometry and the finite reference-tracking law. The central division is simple: the host owns $\mathbb{CP}^{n-1}$; the IOF owns the Δ_n projection available to a finite observer.

Four results carry the construction. Decoherence realizes the modulus map on pointer records. The record geometry is Fisher–Rao, giving the calibrated binary Born form while leaving phase to the host. Exact self-contained reference closure fails by Breuer non-factorization, and approximate tracking obeys the data-rate inequality $\kappa_{\rm op} \geq h_{\rm KS} - C_{\rm eff} \ln 2$. Finally, $h_{\rm KS}$ is an emergent semiclassical branch rate, not an entropy production of a finite closed quantum system.

The resulting picture is conditional but useful. If the BLQC recoverability/sign-reversal tests and the Fisher-homogeneity module [2] survive, the finite-reference control law is physically calibrated within standard quantum mechanics. Conditional on Everett's host-side measure and rational-use account, Everett+IOF then gives a compact no-collapse picture: one unitary substrate, emergent quasi-classical branches, finite branch-local records, hosted Born weights, an independently grounded fixed-context Born form, and a capacity-set cut. What remains open is also clear: phase and cross-context coherence, the ex-ante decision bridge, the host's global structural questions, and the experiments.

A Branch-Record Witness for Exact Non-Factorization

This appendix proves the exact closure obstruction used in Theorem 5.1. It does not prove the rate inequality; that is done in Appendix B.

Lemma A.1 (Factorization criterion). *Let X be a set, $R : X \to Y$ a record map, and $\Theta : X \to Z$ a target functional. There is a function $F : R(X) \to Z$ such that*

$$\Theta = F \circ R$$

if and only if Θ is constant on the fibres of R:

$$R(x) = R(y) \implies \Theta(x) = \Theta(y).$$

Proof. If $\Theta = F \circ R$, equal records clearly give equal Θ-values. Conversely, if Θ is constant on each fibre, define $F(r) = \Theta(x)$ for any x with $R(x) = r$. Fibre-constancy makes this well-defined. □

Proposition A.2 (Two-qubit Breuer witness). *Let $\mathcal{H} = \mathcal{H}_O \otimes \mathcal{H}_W$ and*

$$|\Phi(\theta)\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle_O|0\rangle_W + e^{i\theta}|1\rangle_O|1\rangle_W\big).$$

Then

$$\rho_O(\theta) = \mathrm{Tr}_W\,|\Phi(\theta)\rangle\langle\Phi(\theta)| = \frac{1}{2}I_O$$

for every θ. Thus every record map depending only on the observer's local state gives the same record for all θ, while the global functional $\Theta(\Phi(\theta)) = \theta$ differs.

Proof. Expanding the projector gives two diagonal terms and two cross terms. The partial trace over W kills the cross terms because $\langle 0|1\rangle = 0$, leaving $\frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = I_O/2$. Taking, for example, $\theta = 0$ and $\theta = \pi/2$ gives equal local records and different global Θ-values. Lemma A.1 then forbids exact factorization through the observer's record. □

The witness is static. It shows non-injectivity of the restriction from global state to local record. The rate law enters only when the hidden global variable is replaced by a partially trackable branch-effective reference coordinate.

B The Reference-Closure Inequality

This appendix proves Theorem 5.3. The proof combines Pesin production, Holevo extraction, and an estimation converse along the unstable coordinate.

Assumption B.1 (Branch hyperbolicity)**.** In the semiclassical limit, the branch-conditioned reference dynamics is a C^2, uniformly hyperbolic map T, ergodic for an SRB measure μ. For the scalar statement, θ_t lies along a one-dimensional unstable bundle. The SRB conditional measures on unstable manifolds are absolutely continuous with bounded densities on compact local pieces. Continuous weak decoherence selects the branch and keeps the Wigner description in the classical regime where the Zurek–Paz/Pesin rate is meaningful [29, 30].

Lemma B.2 (Production, Pesin)**.** *Under Assumption B.1, the reference produces distinguishable unstable structure at rate*

$$h_{\mathrm{KS}} = \int \sum_i \lambda_i^+ \, d\mu = \sum_i \lambda_i^+.$$

Proof. This is Pesin's identity for the SRB measure. Ledrappier–Young identifies the relevant equality case with absolutely continuous unstable conditionals, the same property used in the estimation converse [32]. □

Lemma B.3 (Extraction, Holevo)**.** *A finite observer subsystem of dimension d_O can carry at most $\log_2 d_O$ bits of distinguishable record per refreshed use. Therefore*

$$C_{\mathrm{eff}} \le \log_2 d_O, \qquad C_{\mathrm{eff}} \ln 2 \le \ln d_O.$$

Proof. The accessible information through any record POVM is bounded by the Holevo quantity, $\chi \le S(\rho_O) \le \log_2 d_O$. Decoherence exports the record after each use, so this is a per-use ceiling. □

Lemma B.4 (Typical-case estimation converse)**.** *Assume Assumption B.1. At each step the observer receives one symbol from an alphabet of size at most $2^{C_{\mathrm{eff}}}$, produced by any adaptive or randomized encoder. Let $S = (S_1, \ldots, S_n)$ be the record and θ_n the unstable coordinate on the universal cover. If*

$$h(\theta_0 \mid \sigma) = \ln(\sqrt{2\pi e}\, \sigma_{0,\mathrm{eff}})$$

for the transverse fibre label σ, then every estimator $\hat{\theta}_n(S)$ obeys

$$\mathbb{E}[(\theta_n - \hat{\theta}_n)^2] \ge \sigma_{0,\mathrm{eff}}^2 e^{2n(h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2)}.$$

Consequently $\kappa_{\mathrm{op}}(n) \ge h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$ before saturation.

Proof. The record has at most $2^{nC_{\mathrm{eff}}}$ values, so

$$I(\theta_0; S \mid \sigma) \le nC_{\mathrm{eff}} \ln 2.$$

Thus

$$h(\theta_0 \mid S, \sigma) \ge h(\theta_0 \mid \sigma) - nC_{\mathrm{eff}} \ln 2.$$

Along a fixed unstable fibre the dynamics maps θ_0 to θ_n with unstable Jacobian J_n^u. Change of variables gives

$$h(\theta_n \mid S = s, \sigma) = h(\theta_0 \mid S = s, \sigma) + \mathbb{E}[\ln J_n^u \mid S = s, \sigma].$$

Averaging and applying Pesin's identity yields

$$\mathbb{E}[h(\theta_n \mid S, \sigma)] \geq h(\theta_0 \mid \sigma) + n(h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2).$$

The Gaussian maximum-entropy bound gives

$$\mathrm{Var}(\theta_n \mid s, \sigma) \geq (2\pi e)^{-1} e^{2h(\theta_n \mid s, \sigma)}.$$

Since an S-measurable estimator cannot beat conditional variance, Jensen's inequality gives the stated mean-square lower bound. □

Remark B.5 (Why condition on transverse fibres)*.* The conditioning on σ is essential. A volume-preserving map may preserve total differential entropy while expanding unstable directions. The production relevant to tracking is the unstable conditional entropy, not the full phase-space entropy.

Proof of Theorem 5.3. Lemma B.3 gives $\kappa_{\mathrm{def}} = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2 \geq h_{\mathrm{KS}} - \ln d_O$. Lemma B.4 gives $\kappa_{\mathrm{op}} \geq h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$. If $h_{\mathrm{KS}} > \ln d_O$, both the structural deficit and operational growth lower bound are positive. □

Remark B.6 (Inequality scope)*.* The rates h_{KS} and C_{eff} are fixed by the branch map and record channel before any tracking run. The growth of σ_θ^2 is a consequence of their mismatch through Lemma B.4, not an inserted ansatz. The lemma gives only the necessity bound. Turning it into the exact identity $\kappa = h_{\mathrm{KS}} - C_{\mathrm{eff}} \ln 2$ would require a saturating estimator and a generating pointer partition; otherwise the observer sees a lower-entropy projection or loses rate through inefficiency, and the inequality remains strict.

C The Accessible Record Algebra from Broadcast Structure

Proof of Proposition 2.7. In exact SBS, the conditional fragment states have orthogonal supports. Their support projectors commute, generate A_f, and can be read by a projective measurement on the fragment alone.

In the approximate case, use the pretty-good measurement on the ensemble $\{p_i, \sigma_i^{(k)}\}$. The Barnum–Knill bound gives

$$P_{\text{err}} \le \sum_{i \ne j} \sqrt{p_i p_j}\, F(\sigma_i^{(k)}, \sigma_j^{(k)}) \le (n-1)\varepsilon_f .$$

Reading ρ rather than the nearby SBS state σ changes any single POVM probability by at most ε_s. Hence $\delta \le (n-1)\varepsilon_f + \varepsilon_s$. The gentle measurement lemma then bounds the disturbance by $2\sqrt{\delta}$.

Coarse-graining A_f to at most 2^m elements is a deterministic Markov map, giving the observer-indexed algebra A_O. Agreement among several observers follows by the union bound: if each readout fails with probability at most δ, joint agreement fails with probability at most $t\delta$. The exact maximality claim follows from the corrected equivalence between strong quantum Darwinism plus strong independence and SBS form [8, 12–14]. □

The approximate maximality converse is not claimed. The literature has exact SBS equivalence theorems and approach-to-SBS bounds [8, 45]; a general approximate converse is still an open problem in that literature.

D Proofs for Section 8

Proof of Proposition 8.1. Let $\omega_1, \dots, \omega_N$ be the atoms of the finite algebra A. A bet on event E at rate $\beta(E)$ and stake $s \in \mathbb{R}$ pays

$$s(\mathbf{1}_E(\omega) - \beta(E))$$

at atom ω. The set of portfolio payoffs is the linear span of $\mathbf{1}_E - \beta(E)\mathbf{1}$. Weak coherence says this subspace contains no strictly negative vector. By finite-dimensional separation, there is a nonzero $\lambda \geq 0$, normalized by $\sum_\omega \lambda_\omega = 1$, orthogonal to every generator. Hence

$$\beta(E) = \sum_{\omega \in E} \lambda_\omega.$$

So β is a finitely additive normalized measure. The converse is immediate because every portfolio has zero expectation under that measure and therefore cannot be uniformly negative. Tower-consistency is exactly the statement that these measures push forward under sufficient coarse-grainings. □

Proof of Proposition 8.6. Consider a post-branching, pre-registration state

$$|\Psi\rangle = \sum_k \alpha_k |k\rangle_S |E_k\rangle |R_0\rangle_O,$$

with decohered but record-identical alternatives.

If two branches have equal moduli, a unitary swap acting outside the accessible record algebra preserves the observer's record state. By the record-credence principle the credence assignment is unchanged; by relabeling symmetry the two credences exchange. Hence they are equal. Normalization gives equal weights for equal-modulus branches.

If $|\alpha_k|^2 = m_k/n$ is rational, fine-grain branch k into m_k equal-weight sub-branches. Equal-weight symmetry gives each shard credence $1/n$, and tower-consistency pushes this back to $c_k = m_k/n$.

For general weights, approximate them by rational fine-grainings within a single configuration: take $m_k = \lfloor w_k N \rfloor$ equal shards and a remainder of weight less than $1/N$. Equal shards have common credence; nonnegativity bounds the total remainder credence by a vanishing amount. Letting $N \to \infty$ gives $c_k = w_k = |\alpha_k|^2$. On calibrated contexts these are the same weights obtained from the record-geometric construction. □

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